M.Sc. - I (Mathematics) (NEP Pattern) Semester-I

NEP-62 - DSC-2 - Topology Paper-II

P. Pages : 2 **GUG/S/25/15113**

Time : Three Hours * 2 7 5 7 * Max. Marks : 80

Notes : 1. Solve all the **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Prove that every infinite set contains a denumerable subset. **8**

b) Prove that $2^a > a$ for every cardinal number a. **8**

OR

c) Show that the set of all real numbers is uncountable. **8**

d) Prove that **8**

i) $\aleph_0 \aleph_0 = \aleph_0$ ii) $\aleph_0 C = C$ iii) $CC = C$ where N denotes Hebrew letter aleph.

UNIT – II

2. a) Find $d(\{b\})$ & $d(X)$ for $X = \{a, b, c\}$ & $J = \{\phi, X, \{c\}, \{a, c\}, (b, c\}\}$. **8**

b) Let (X, J) be a topological space, $X^* \subseteq X$. Then show that J^* is a topology for X^*, where J^* is induced or relative topology for X^*. **8**

OR

c) Prove that for any set E in topological space, $C(E) = E \cup d(E)$. **8**

d) Show that for any set E in topological space (X, J), $i(E) = \left[C\left(E^C\right)\right]^C$ **8**

UNIT – III

3. a) Prove that a compact subset of a topological space is countably compact. **8**

b) If C is connected set & $C \subseteq E \subseteq C(C)$ then prove that E is a connected set. **8**

OR

c) If f is a homeomorphism of X only X* then prove that f maps every isolated subset of X onto an isolated subset of X*. **8**

d) Prove that a mapping F of X into X* is open iff $F(i(E)) \subseteq i^*(f(E))$ for every $E \subseteq X$. **8**

UNIT – IV

4. a) Prove that in a Hausdorff space, a convergent sequence has a unique limit. **8**

b) Prove that a topological space X is To-Space iff the closures of distinct points are distinct. **8**

OR

c) Prove that in a second axiom space, every collection of nonempty, disjoint open sets is countable. **8**

d) Prove that a topological space X satisfying the first axion of countability is a Hausdorff space iff every convergent sequence has a unique limit. **8**

5. a) Prove that every infinite set is equipotent to a proper subset of itself. **4**

b) Define closure of a set & closed set. **4**

c) Define closed mapping & compact set. **4**

d) Define first axiom space & second axiom space. **4**

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